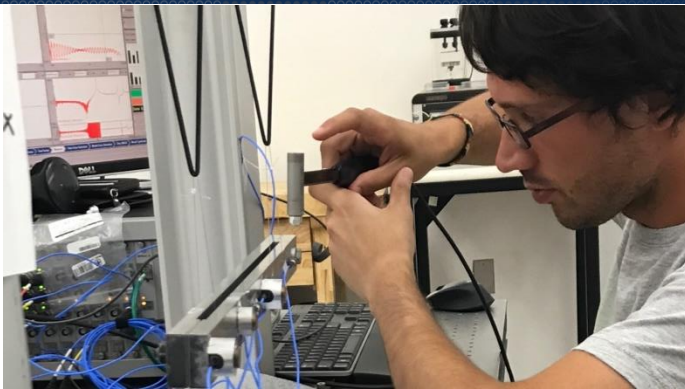
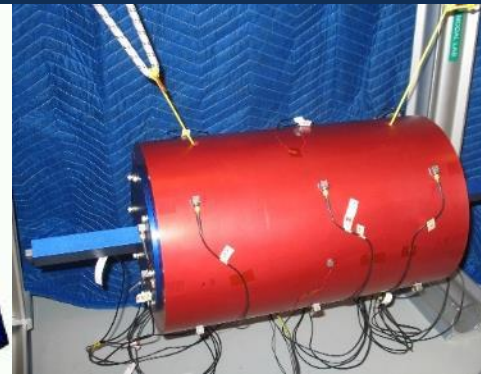
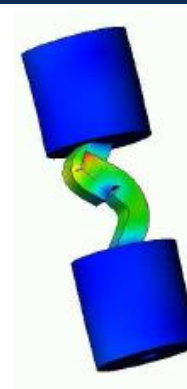


Exceptional service in the national interest



N=O=MAD



Influences of Modal Coupling on Nonlinear Modal Models

Aabhas Singh

Phil Thoenen

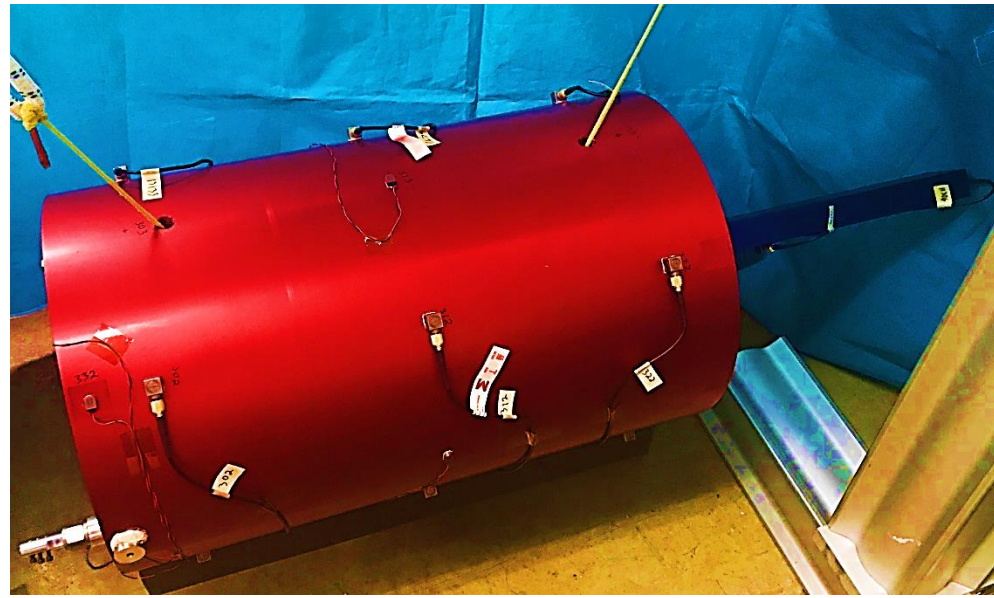
Ben Moldenhauer



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Agenda

1. Introduction
2. Project Overview
3. Experimental Methodology
4. Contact Analysis
5. Nonlinear Parameter Characterization
6. Conclusion



Research Team

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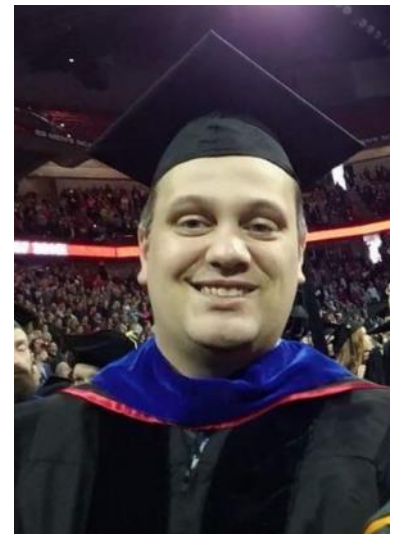
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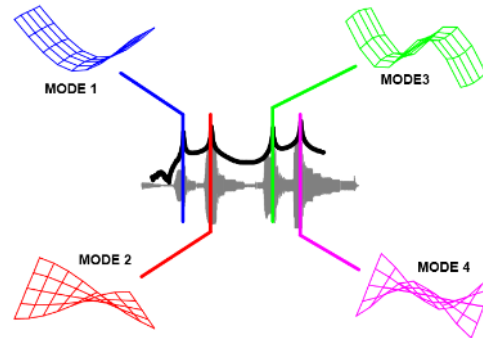


Ben Pacini
Sandia National
Laboratories



Motivation for Modal Analysis

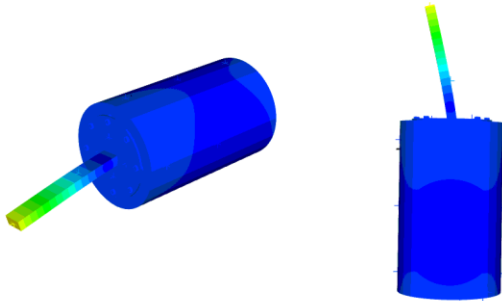
Characterize dynamics of a system under vibrational excitation



SEM Experimental Techniques - February 1998, P. Avitabile

Determine system natural frequencies and mode shapes

Impact design decisions to avoid failure



Wonderfulengineering.com

Linear vs. Nonlinear Systems

- Linear analysis assumes

- Amplitude independent modes
- Modes can be superimposed due to their orthogonality
- Small deformations
- Equation:

$$\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + \omega_r^2q_r = \Phi^T F_{ext}$$

- Psuedo – Nonlinear analysis assumes

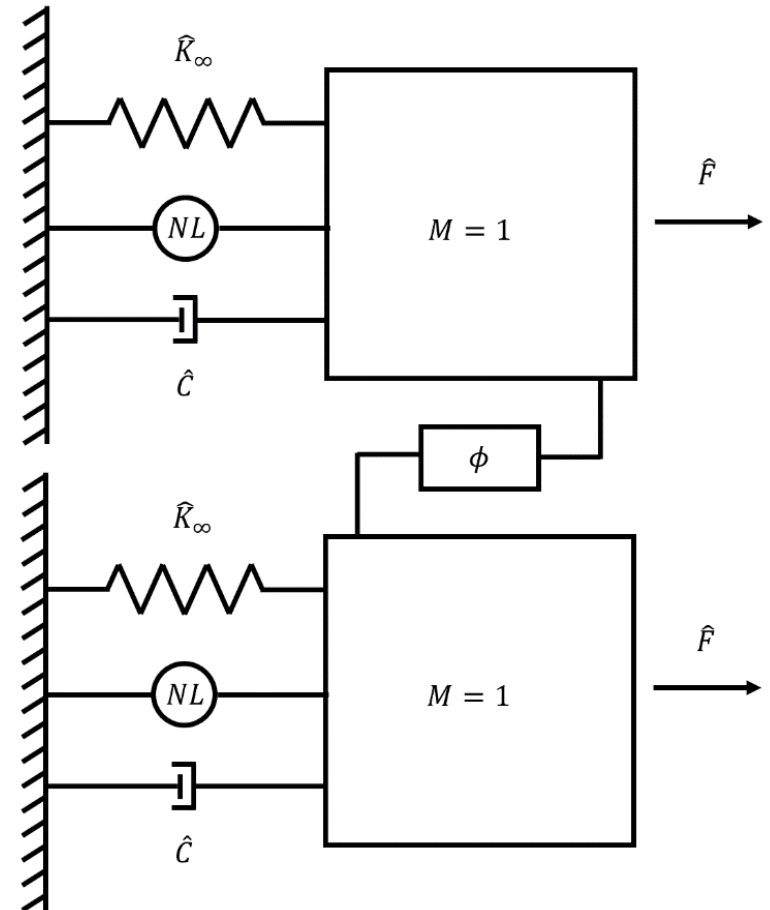
- Linear modes can decouple nonlinear data
- Little to no coupling between modes
- No energy transfer between modes
- Shapes of the linear modes are preserved
- Equation:

$$\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + \omega_r^2q_r + F_{nl}(q_r, \dot{q}_r) = \Phi^T F_{ext}$$

What happens if there is coupling of the modes?

What is modal coupling?

- When the excitation of one mode causes a transfer of energy that perturbs another mode
- Usually occurs due to interactions at joints shared by the different mode shapes



Objectives

Determine the influences of modal coupling on nonlinear modal models

Excite different combinations of modes on a nonlinear structure

Experimentally identify the presence modal coupling

Create a reduced order nonlinear modal model to match experimental results

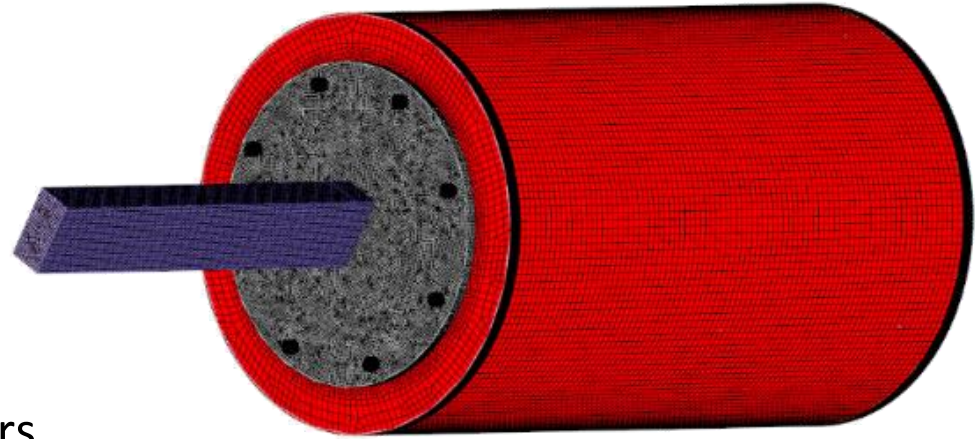


Test System

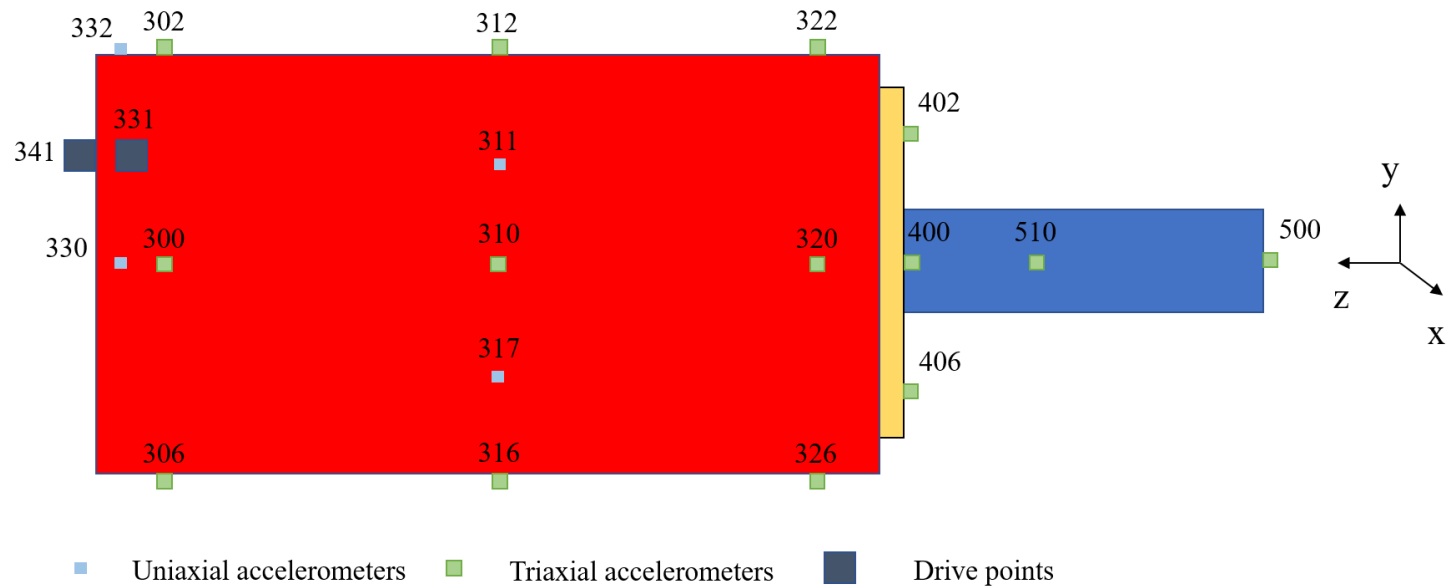
Cylinder – Plate – Beam (CPB)

Plate bolted to cylinder

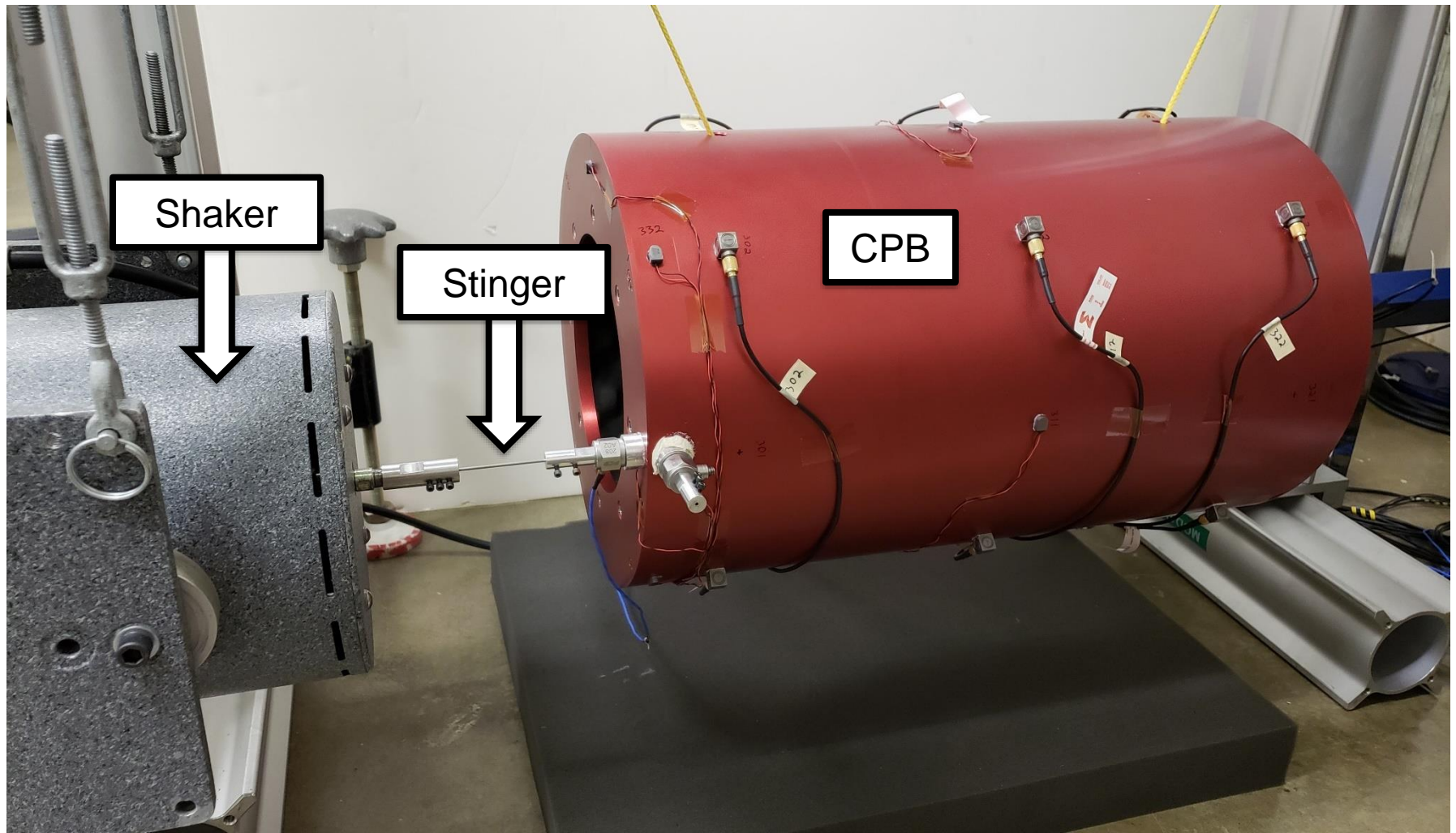
Beam bolted and glued to plate



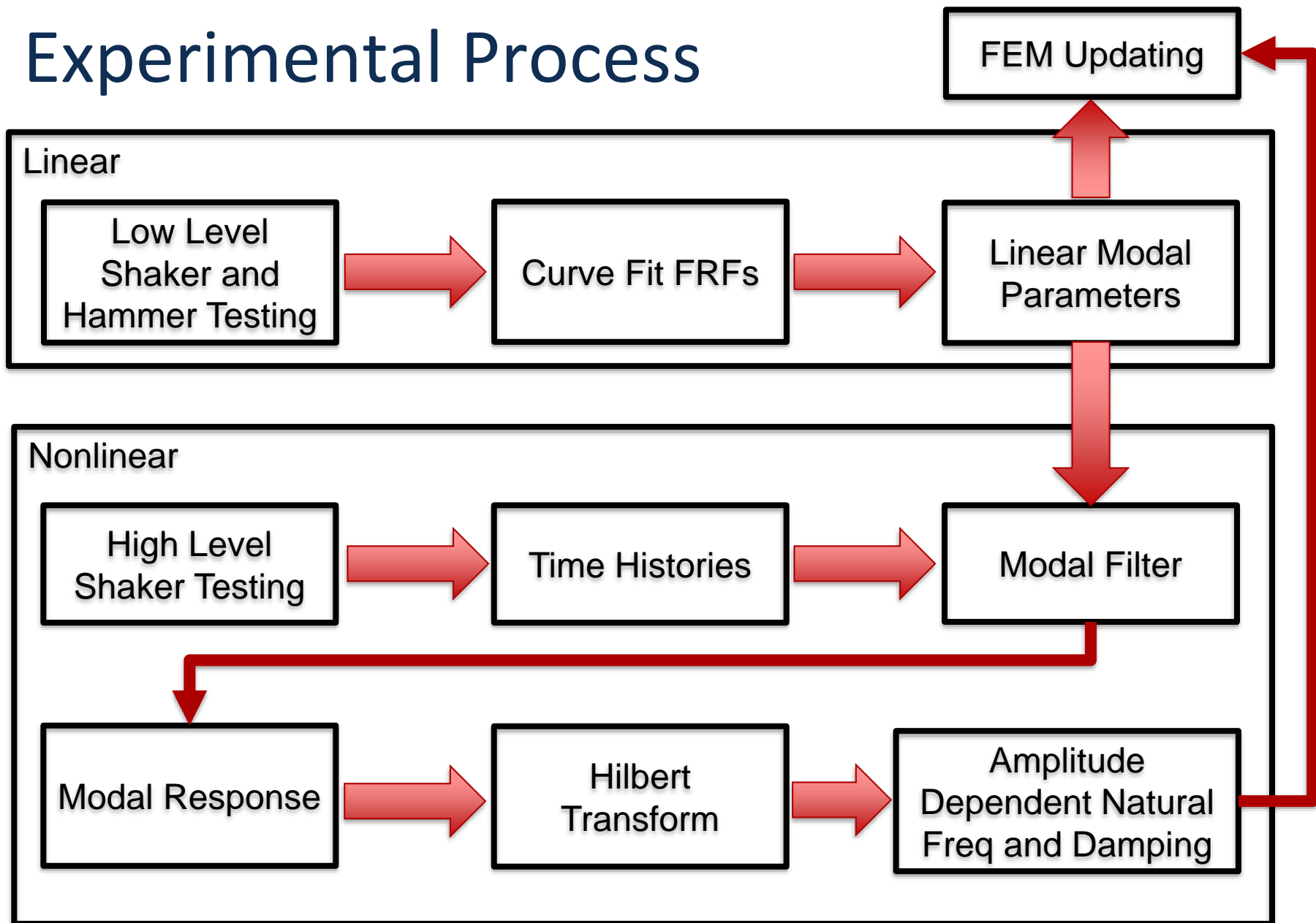
18 triaxial + 8 uniaxial accelerometers



Experimental Setup

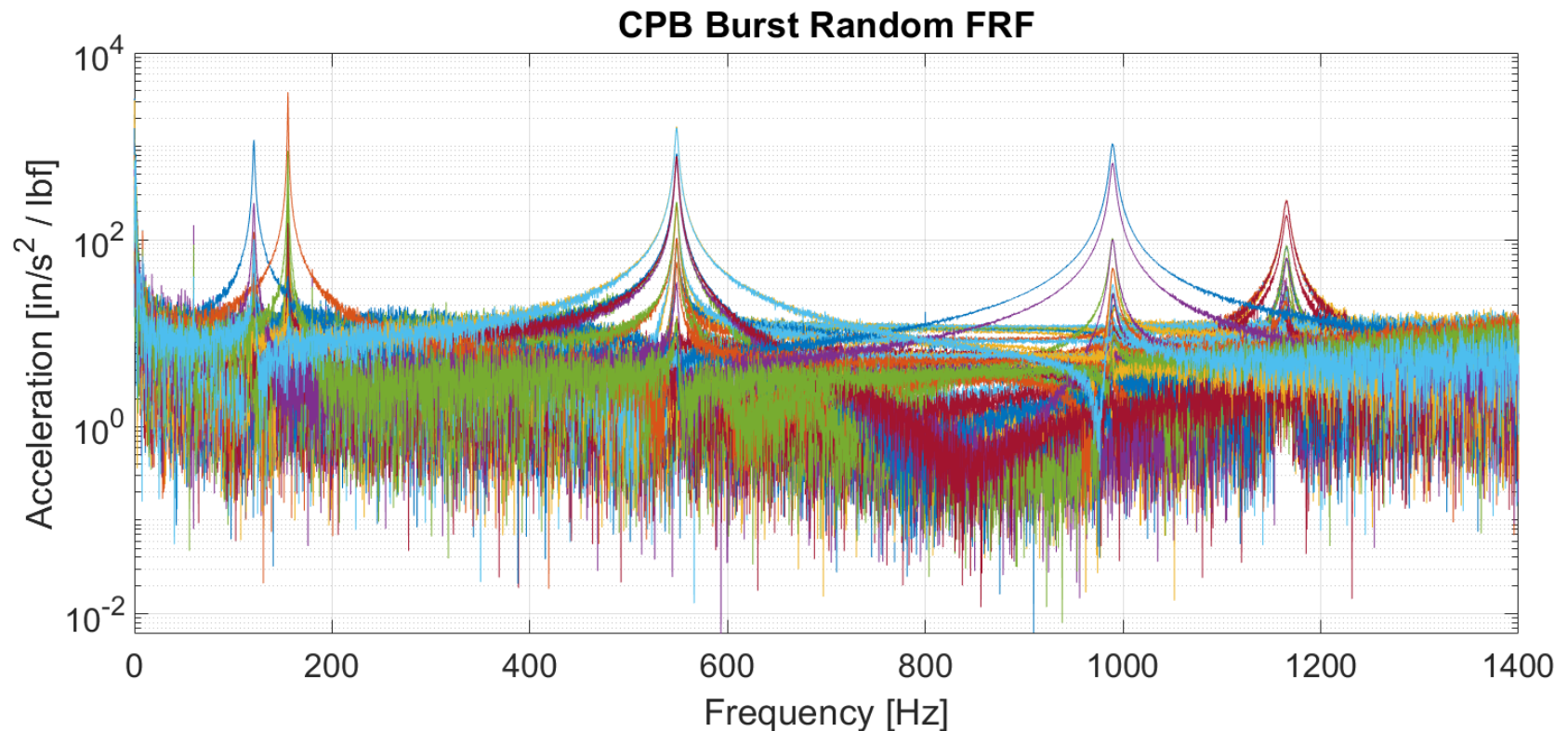


Experimental Process



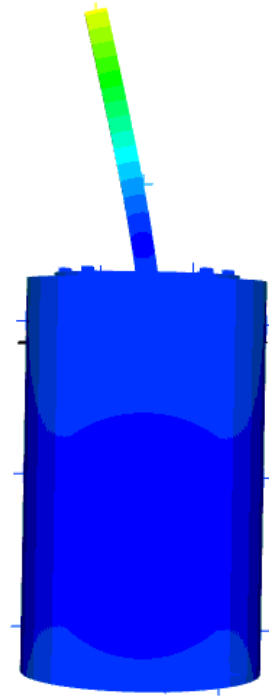
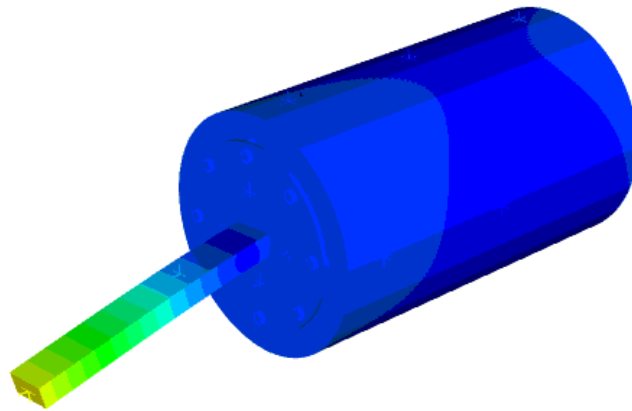
Linear Experimental Data

- Beam bending modes from low level burst random shaker and cylinder modes from light hammer hits
 - Natural frequencies for model updating and shapes for modal filtering



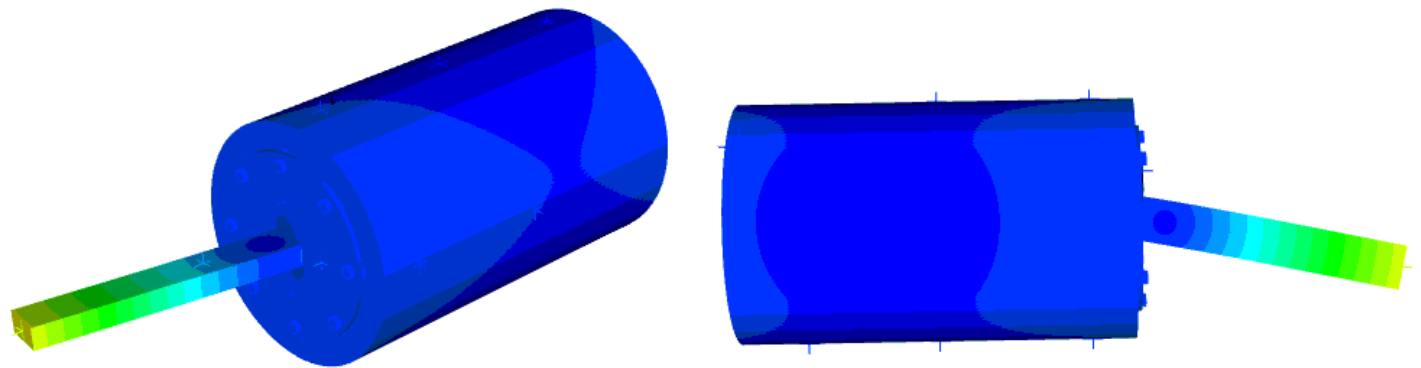
Mode 1

Mode Description	Experimental ω_n (Hz)
1st Beam Bending X	120.8



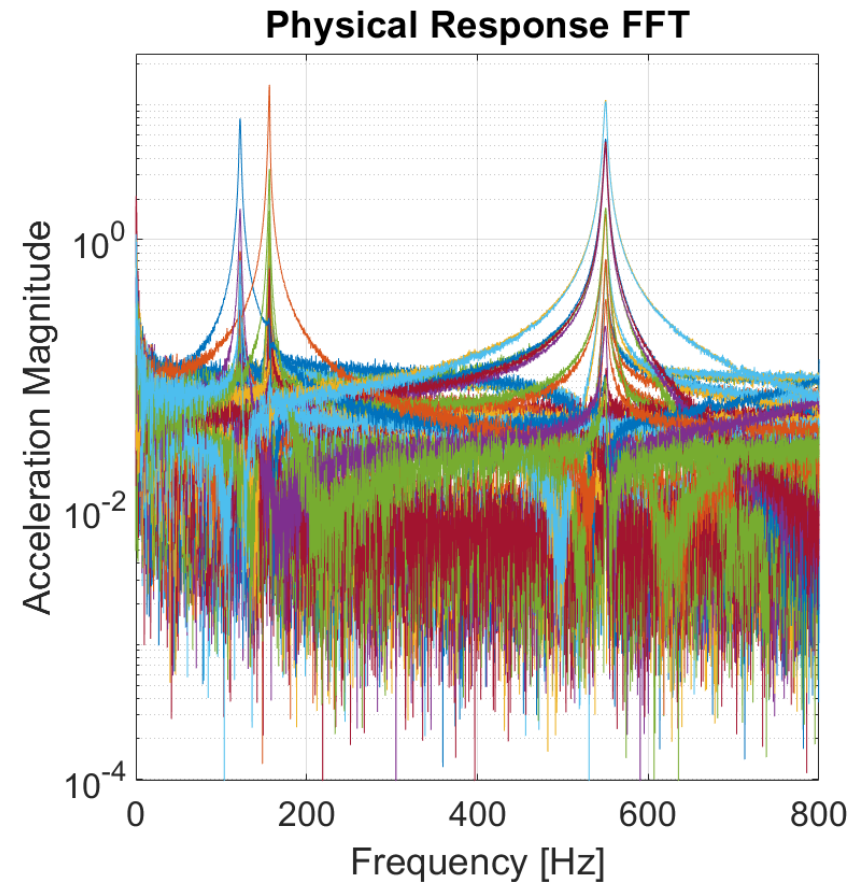
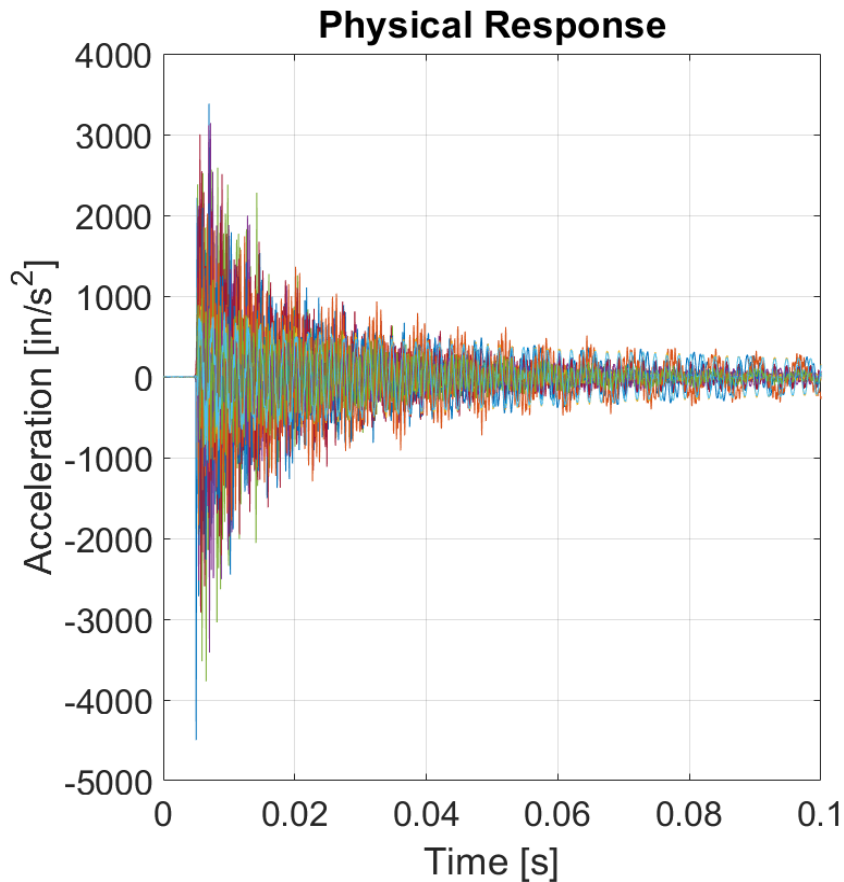
Mode 2

Mode Description	Experimental ω_n (Hz)
1st Beam Bending Y	155.3



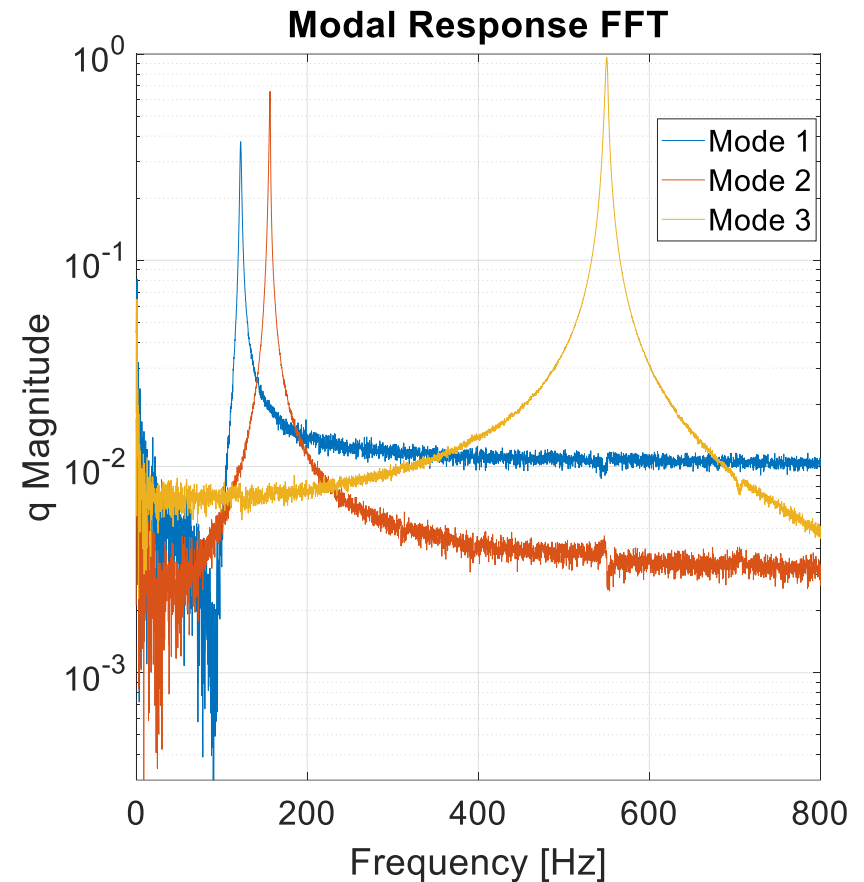
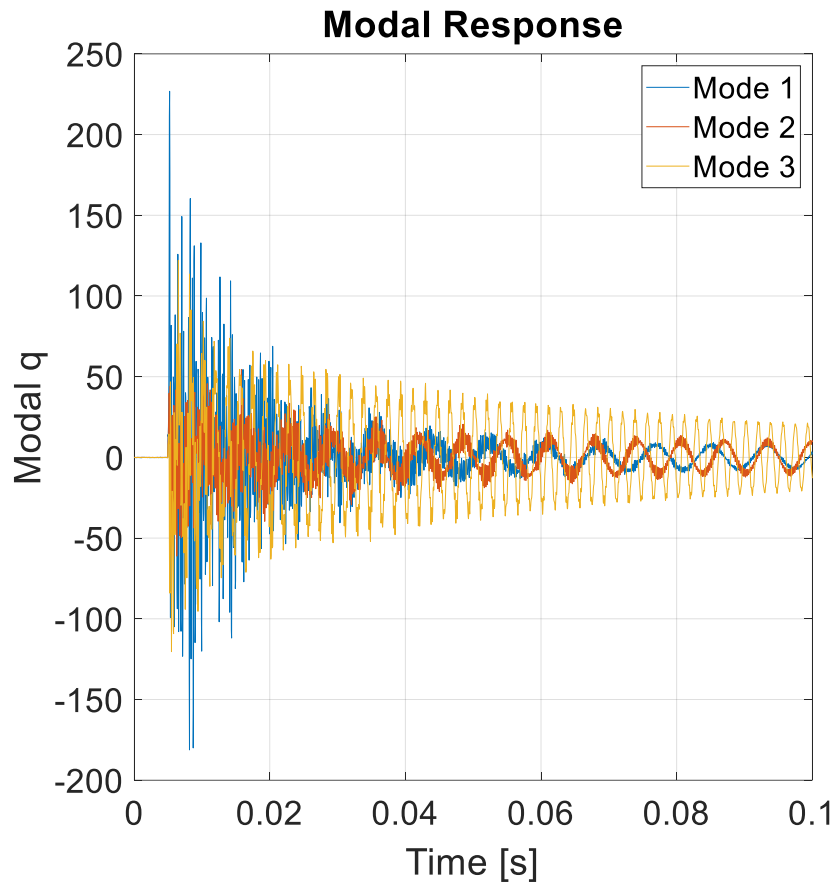
Modal Filtering

- Linear mode shapes allow for filtering of physical response into modal coordinates



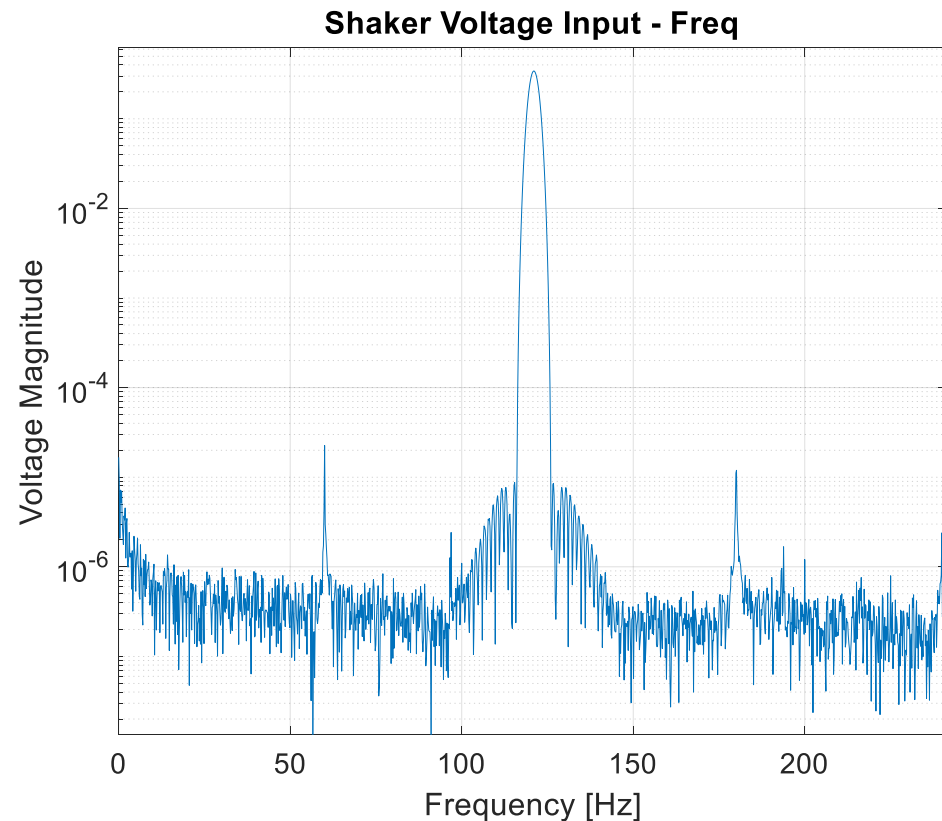
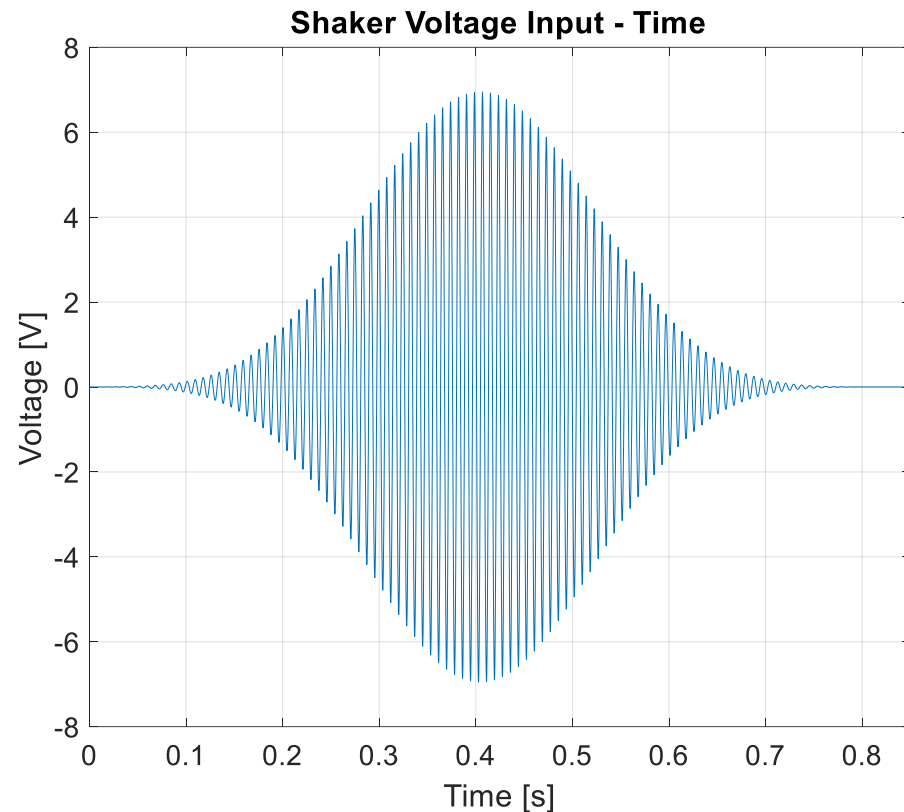
Modal Filtering

- Linear mode shapes allow for filtering of physical response into modal coordinates



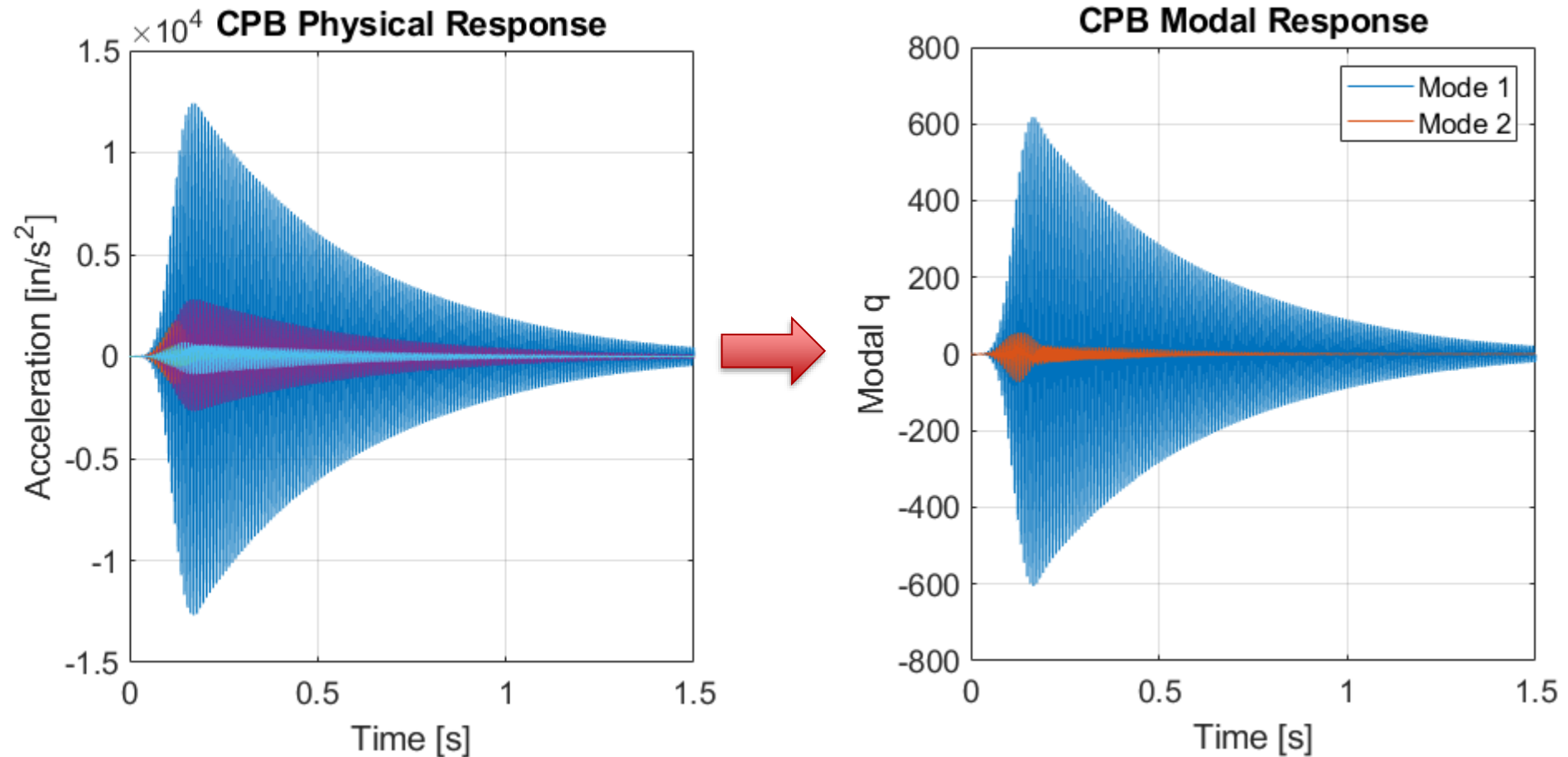
Nonlinear Data

- Shaker delivers definable force input – able to create a voltage signal with specific frequency content



Nonlinear Data

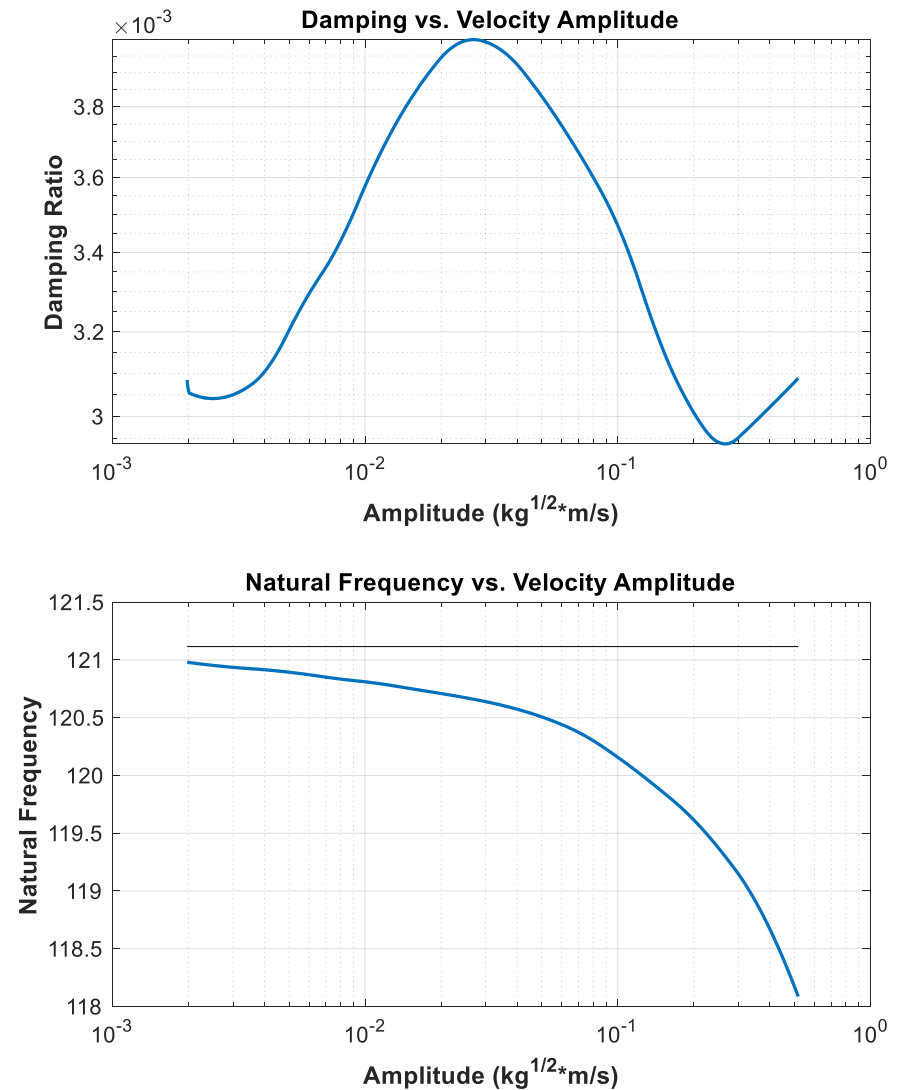
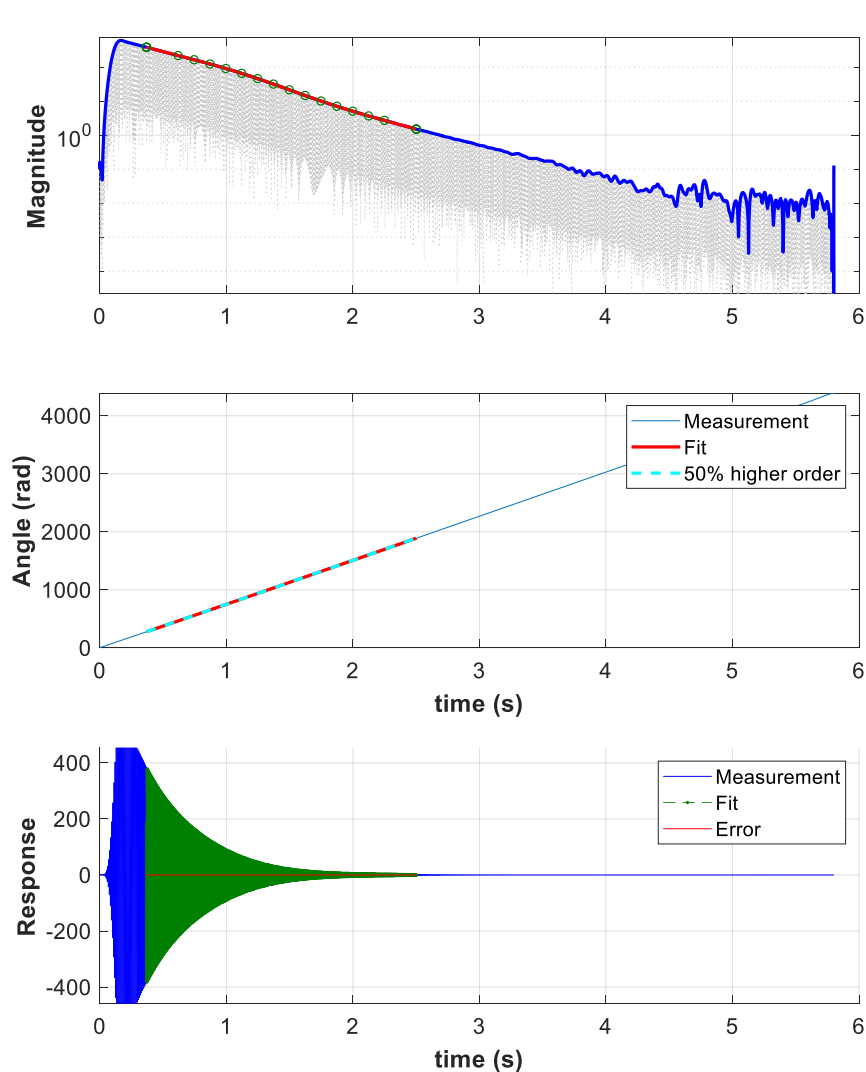
- Use shaker to excite specific modes

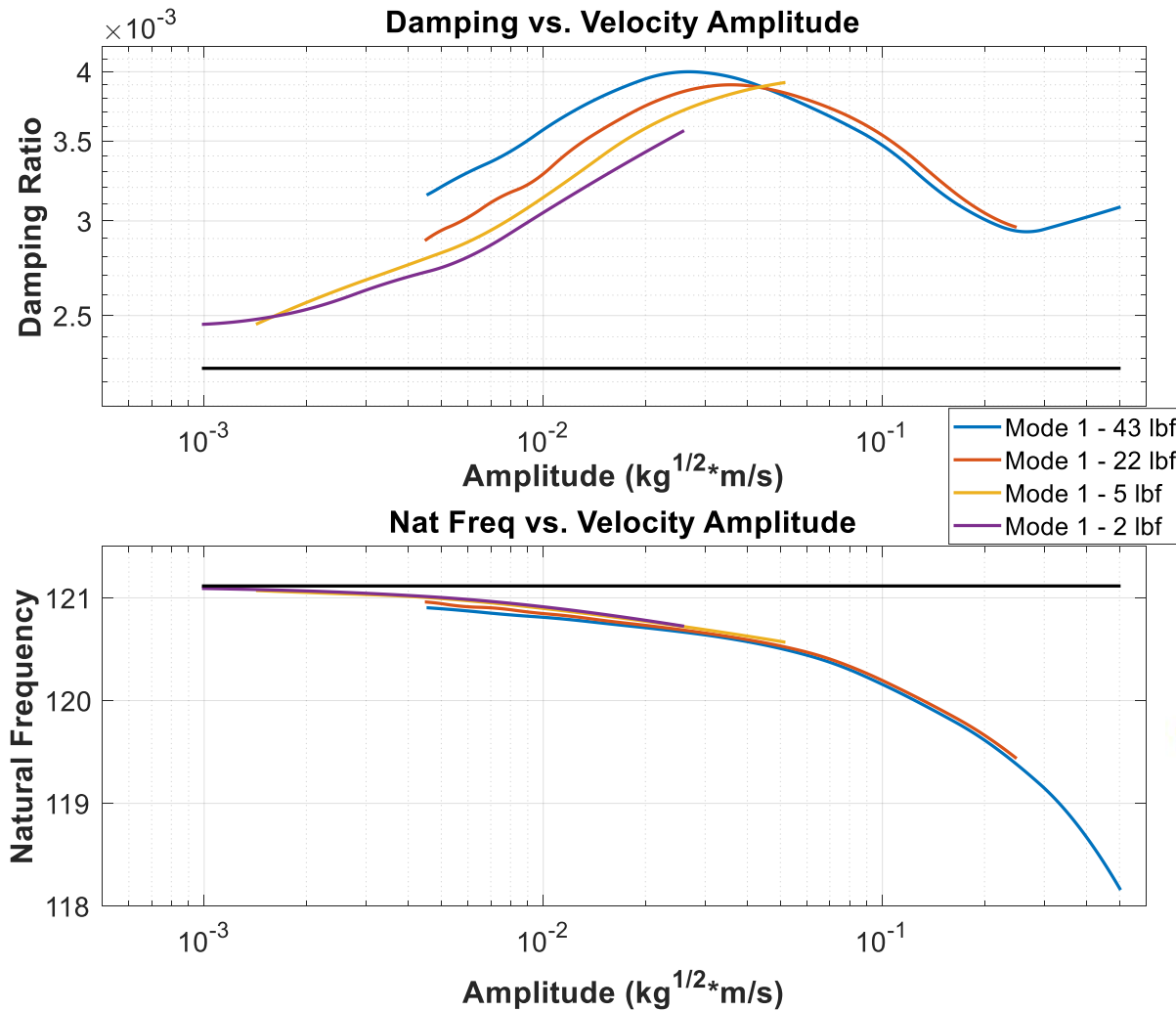


Hilbert Analysis

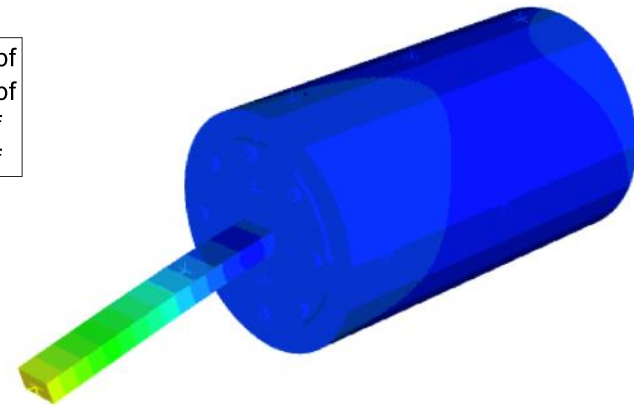
- Requires that each response be uncoupled such that it can be represented by a SDOF system
 - Signal can be represented by a decaying harmonic
 - $\ddot{\eta} = \text{Re}[\exp(\psi_1(t) + i \psi_2(t))]$
- Compute Hilbert Transformation ($\mathcal{H}(t)$) for an amplitude dependent representation of damping and frequency
- $\omega_{d,r} = \frac{d\psi_2}{dt}$
- $\zeta_r \triangleq \frac{d\psi_1}{dt} / \omega_r$

Hilbert Analysis

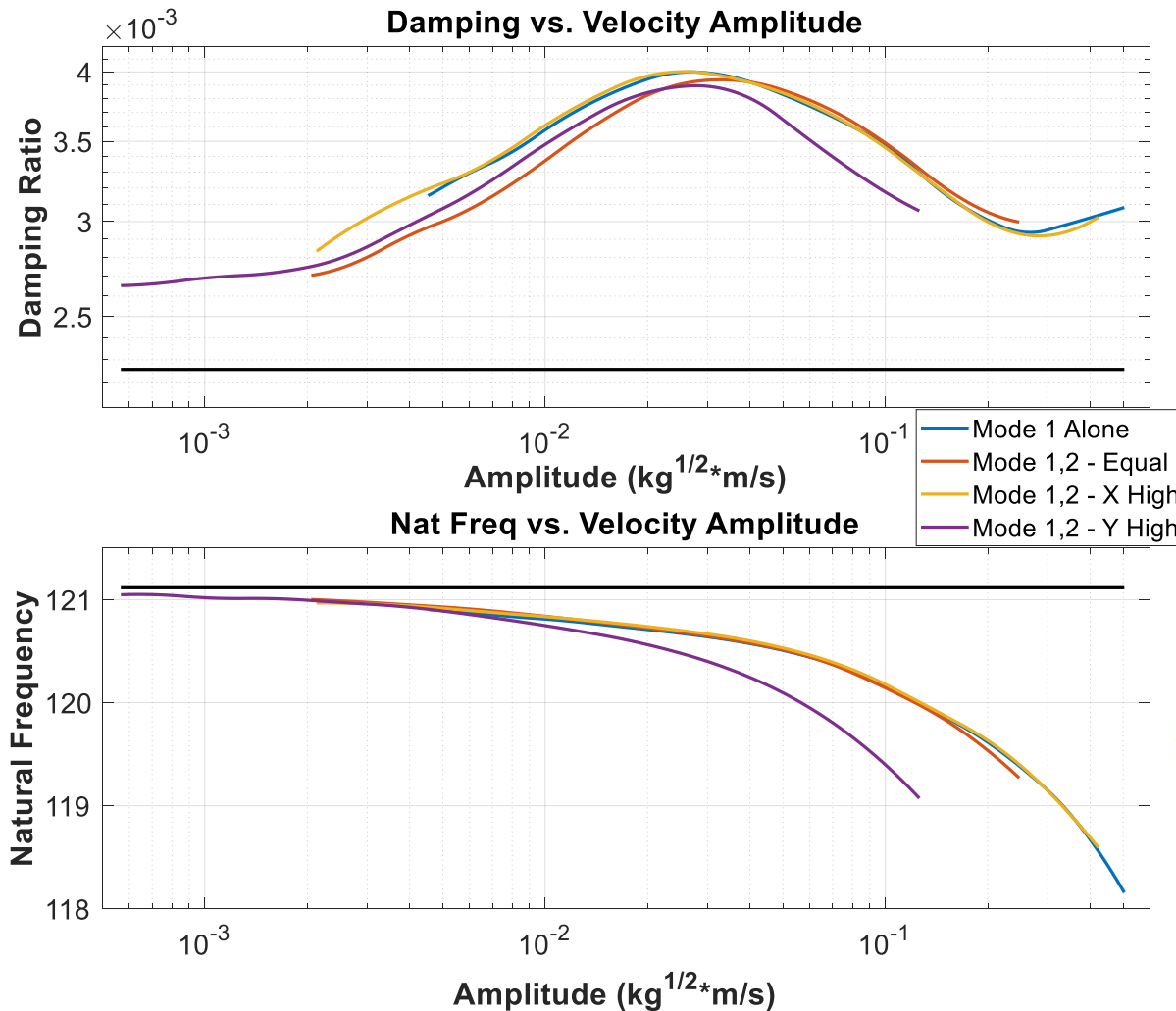




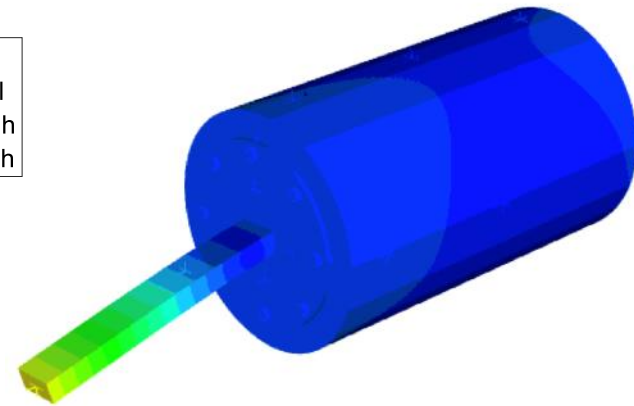
Mode 1



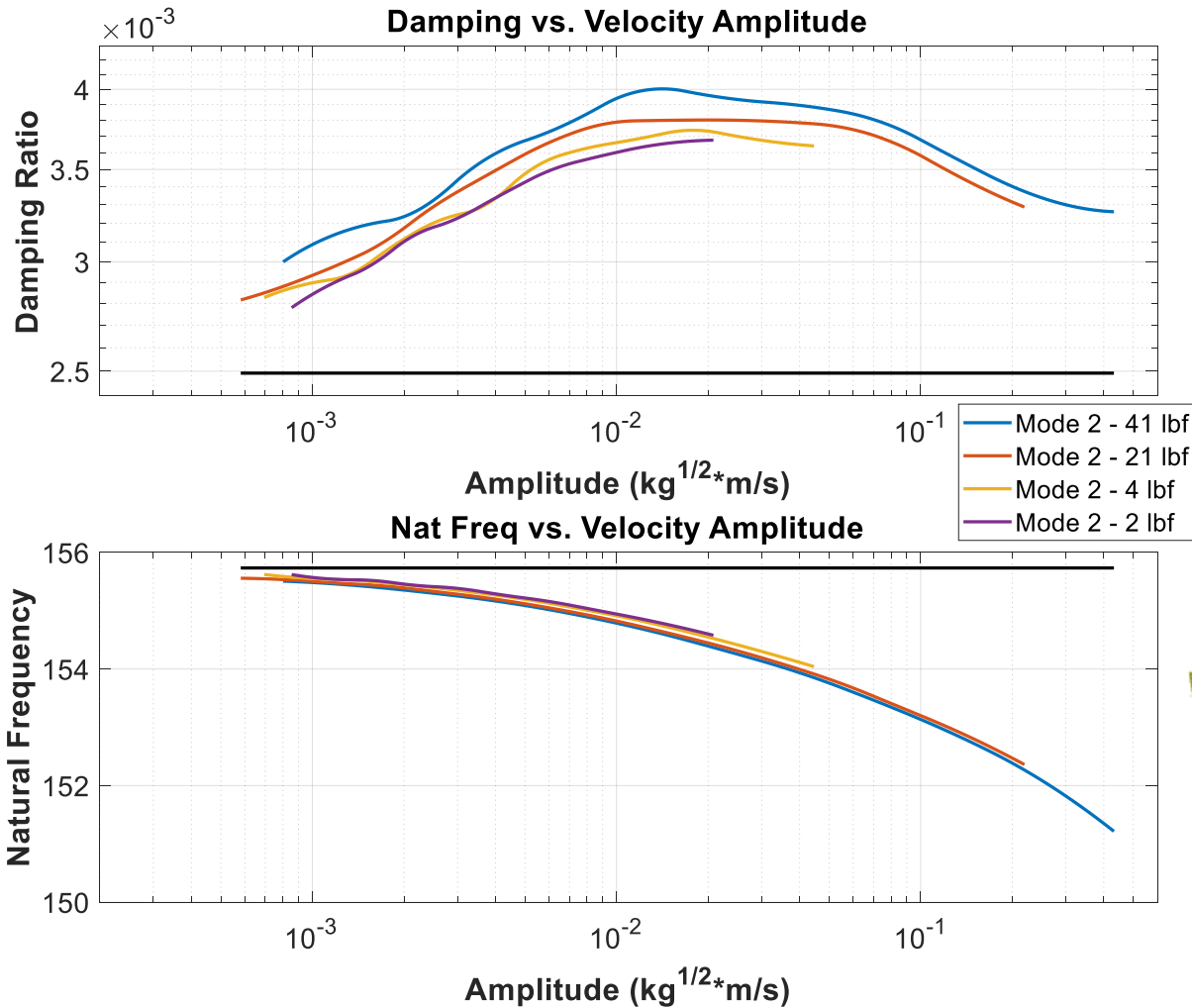
- When excited alone at various levels, frequencies overlay and damping appears to increase with increasing energy



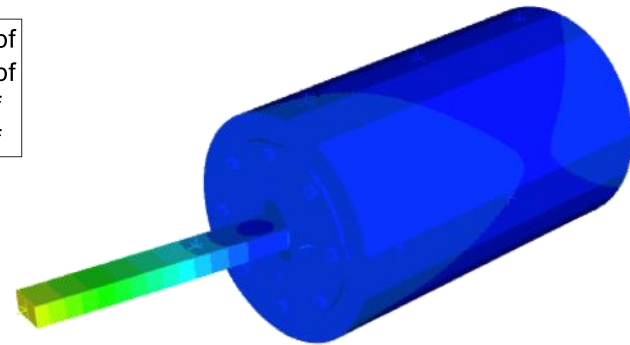
Mode 1



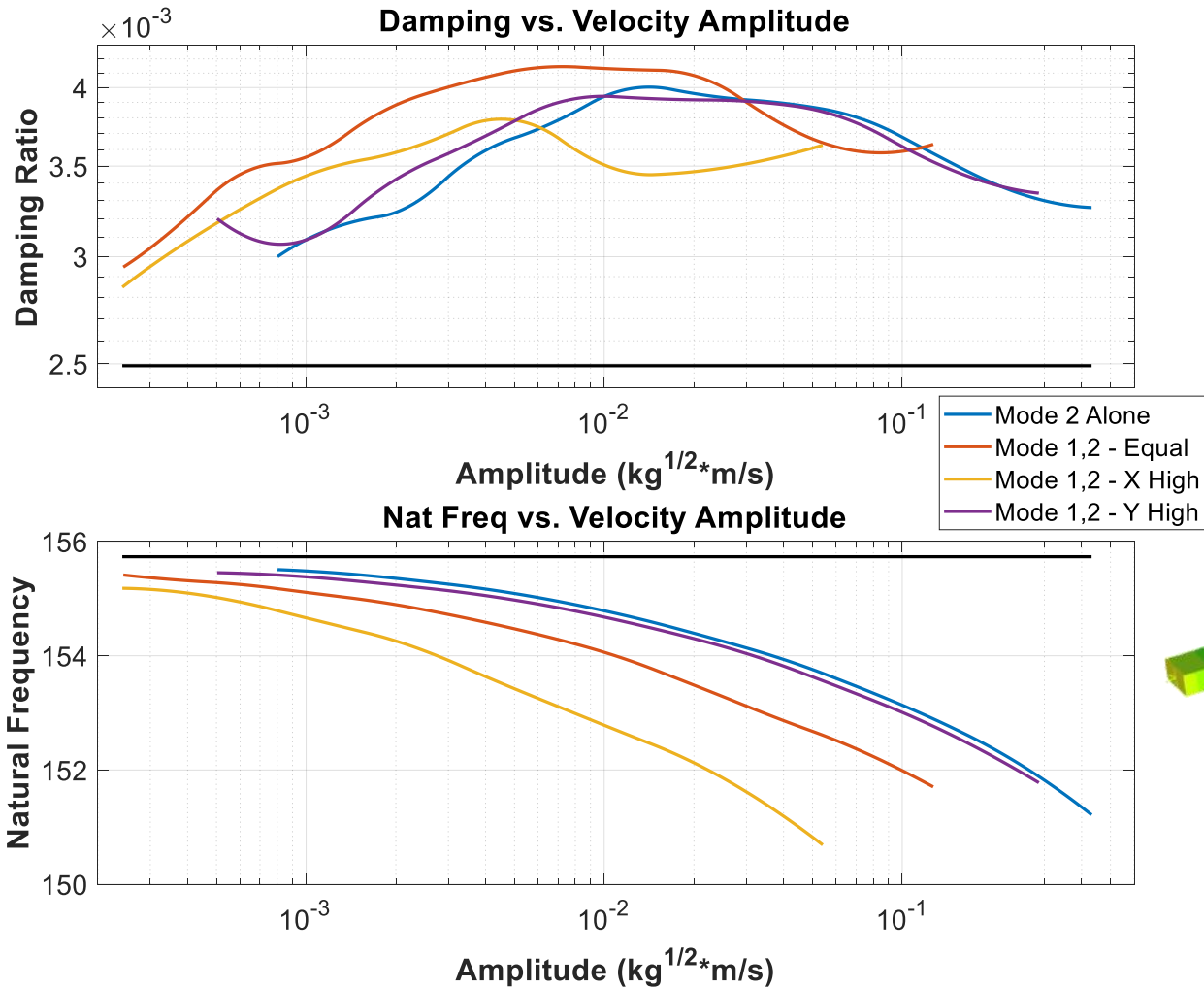
- Coupling visible as a frequency and damping shift when mode 2 is excited to a higher level than mode 1



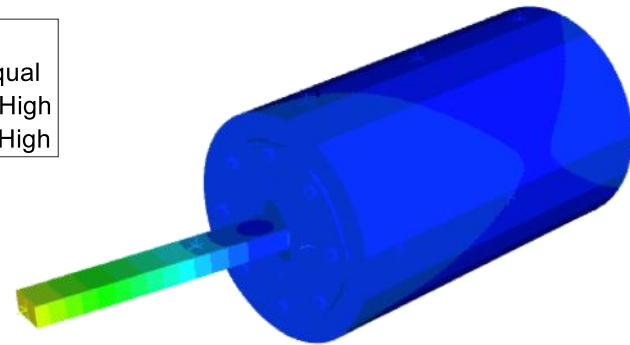
Mode 2



- As with Mode 1, when Mode 2 is excited alone, the frequencies overlay and damping increases with force.



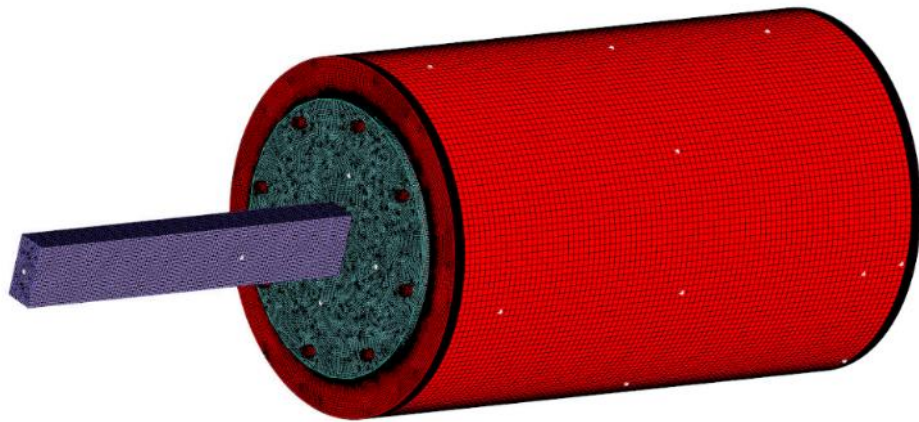
Mode 2



- See same frequency and damping shift, but now to some degree in all cases where mode 1 is also excited.

Model formulation

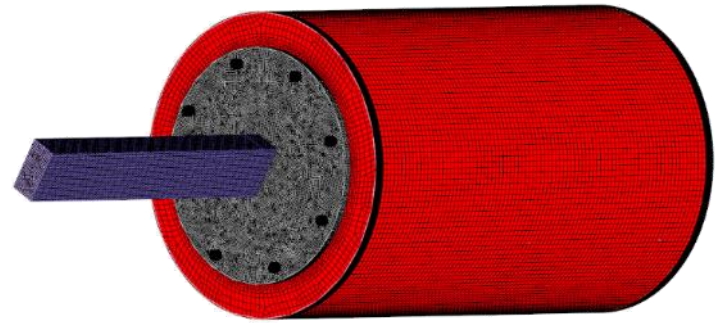
- Developed high fidelity model accounting for entire system geometry
- Updated material properties to match system linear frequencies



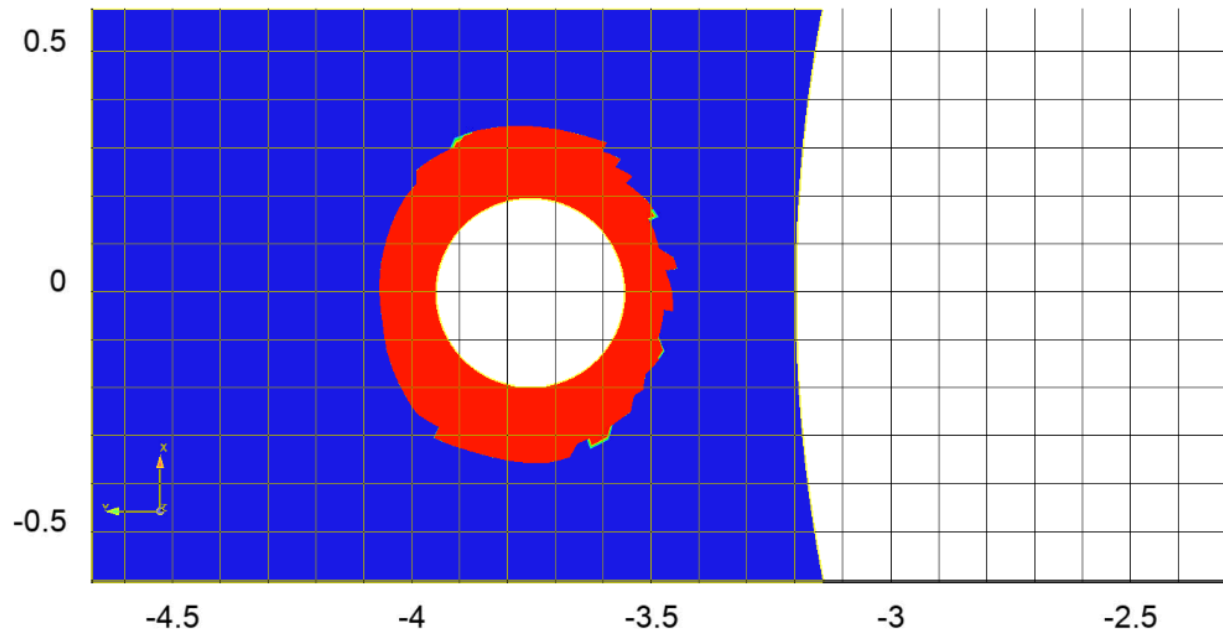
Aluminum	
E_{avg}	10.23 E+6 psi
ν_{avg}	0.34
ρ_{avg}	$2.57 \frac{lbs-s^2}{in^4}$



Modeling Contact Area



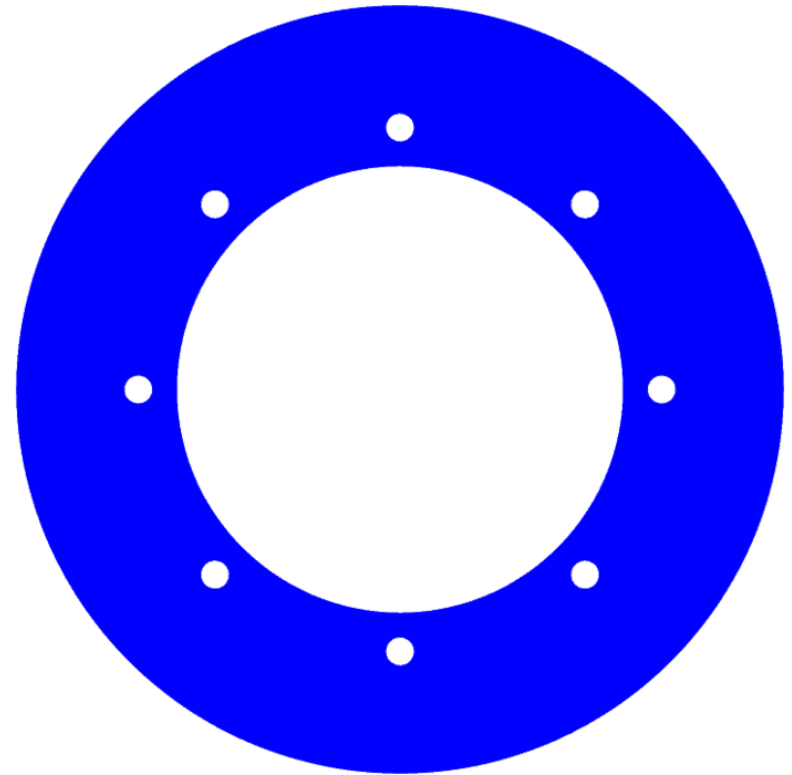
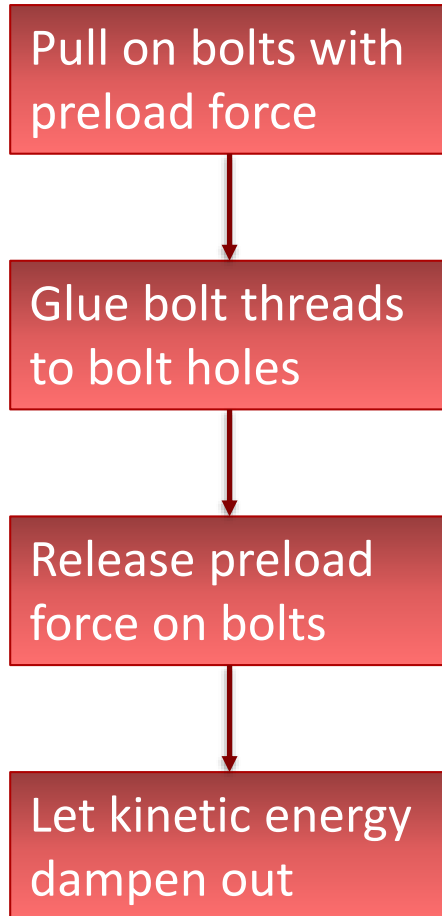
- Bolted structures exhibit slip at the edge of its contact patch
 - This causes hysteresis and an increase in damping
- Primary sources of nonlinearity in the system
 - Opening and closing of the gap between plate and cylinder
 - Joints



Red = contact
Blue = not in contact

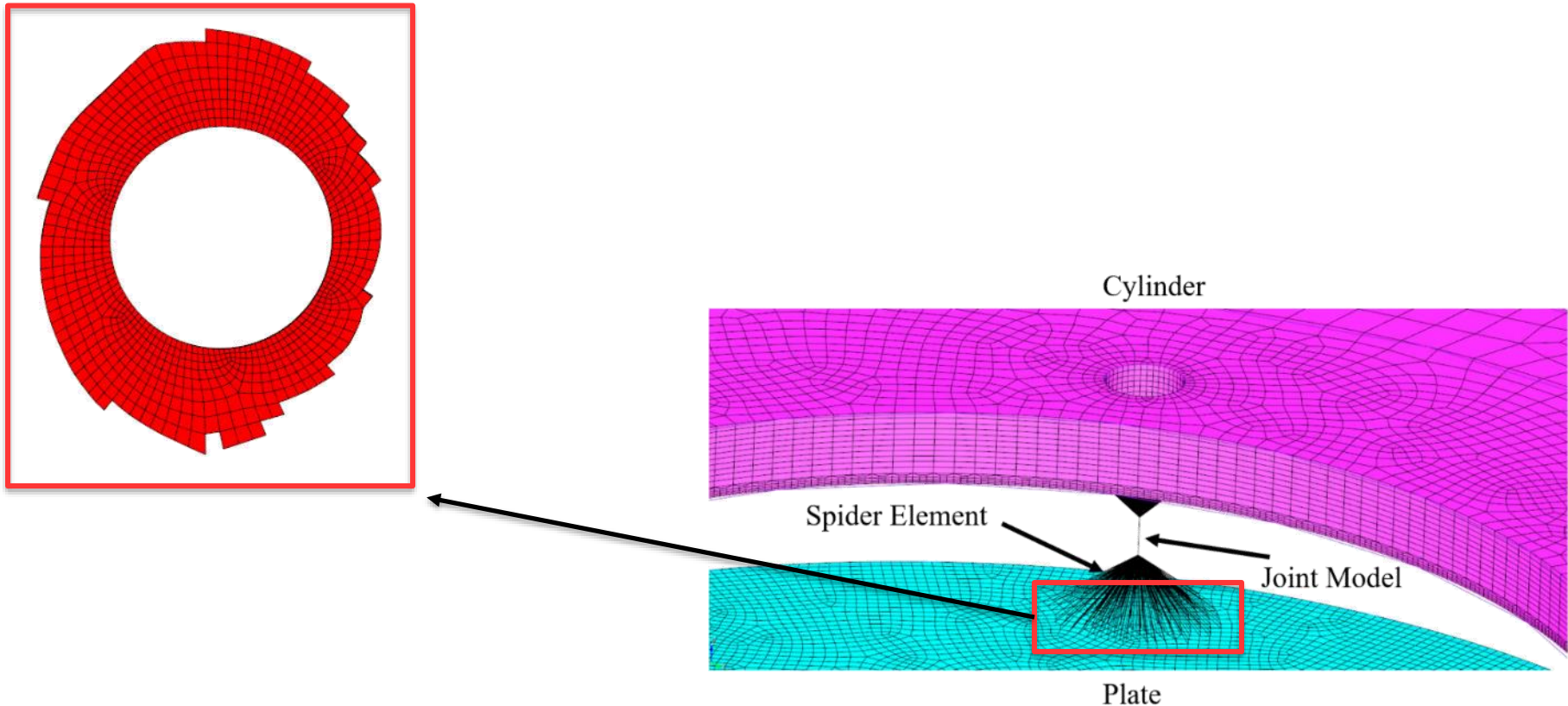
Contact Area

Red = contact
Blue = not in contact

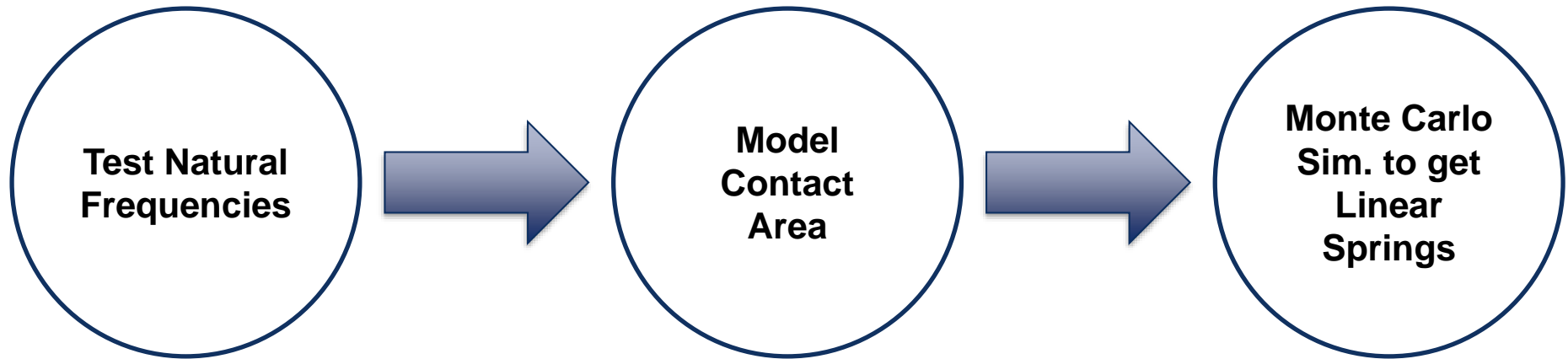


Contact Area to Spidering

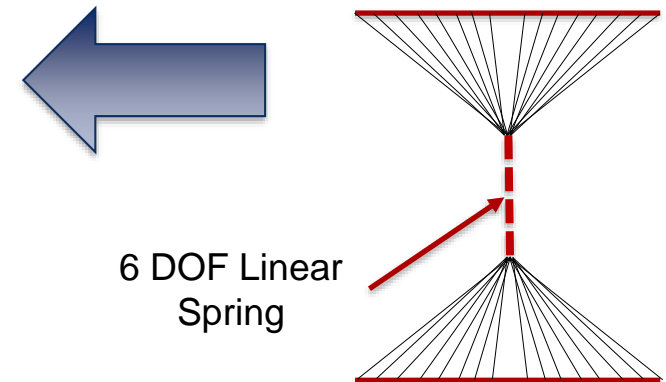
- Spider elements attached to extracted nodes from contact area simulation



Linear Updating

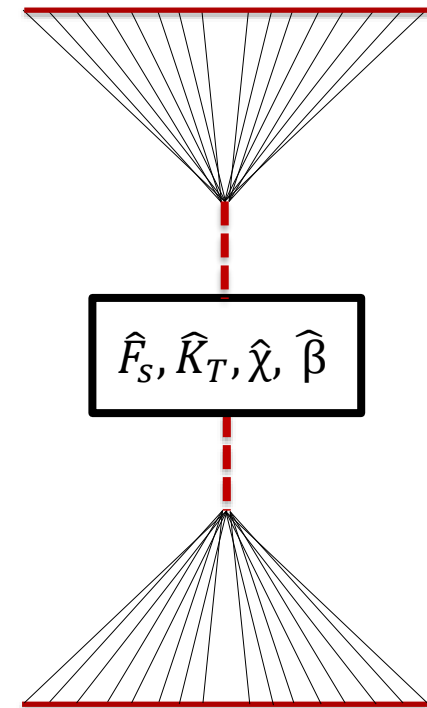


Mode	Experimental f_n [Hz]	Updated Model f_n [Hz]	Percent Error
1	120.8	120.1	-0.63
2	155.3	154.5	-0.56
3	548.4	548.7	0.04
4	989.5	967.9	-2.18
5	1165.1	1168.8	0.32
6	1165.6	1170.4	0.41

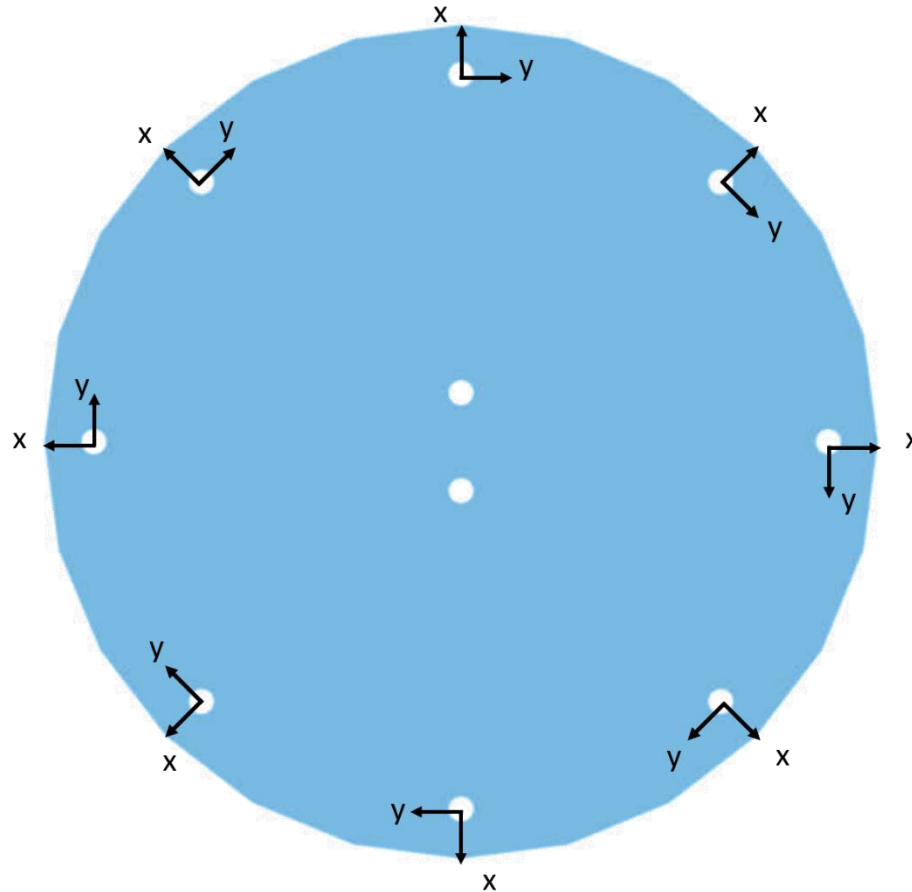


Iwan Parameterization

- Constitutive joint model to describe metal elasto-plasticity behavior
- Each Iwan Joint is comprised of four physical parameters
 - F_S Force required to cause slip
 - K_T Joint stiffness when no slip occurs
 - χ Exponent describing the slope of energy – dissipation curve
 - β Shape parameter of the energy – dissipation curve near macroslip



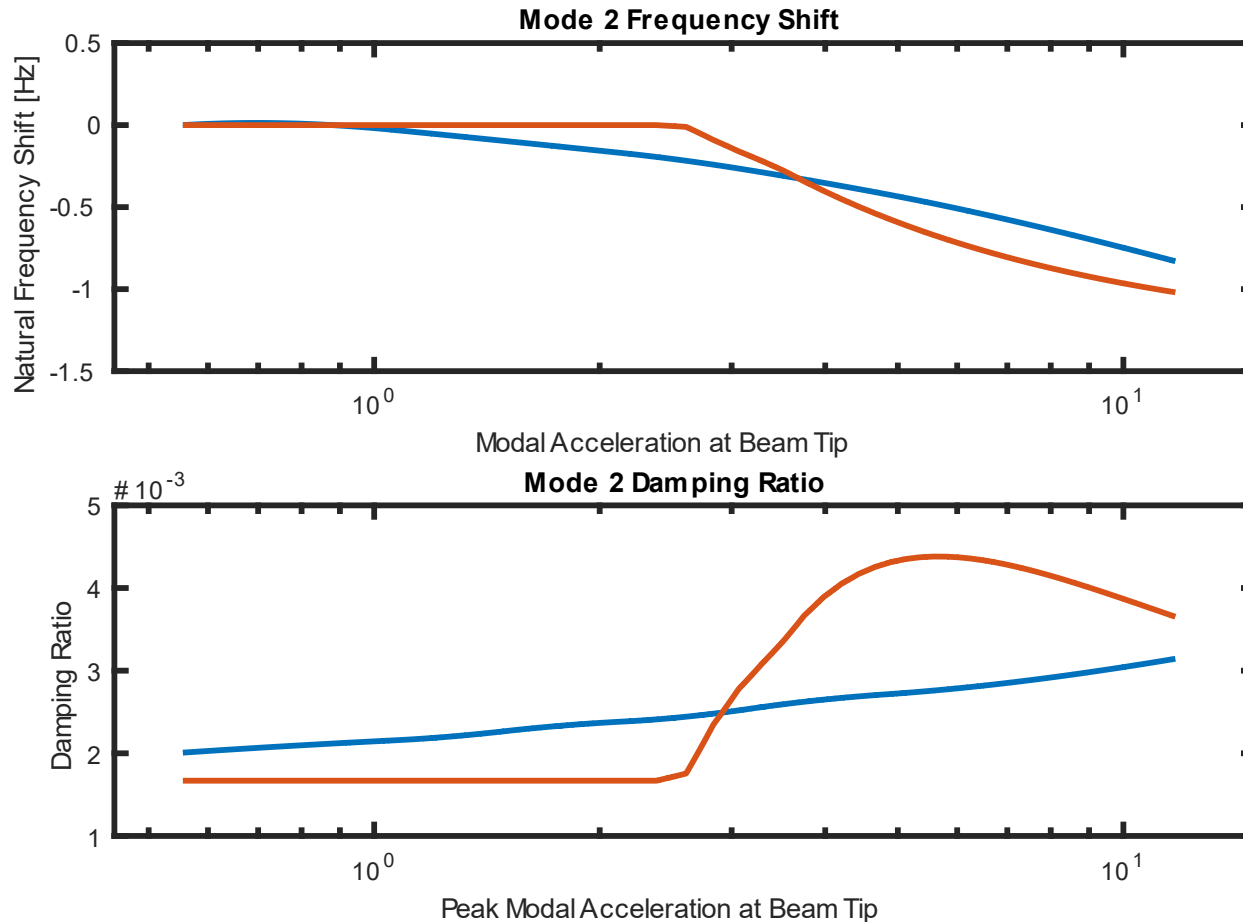
Iwan Parameterization



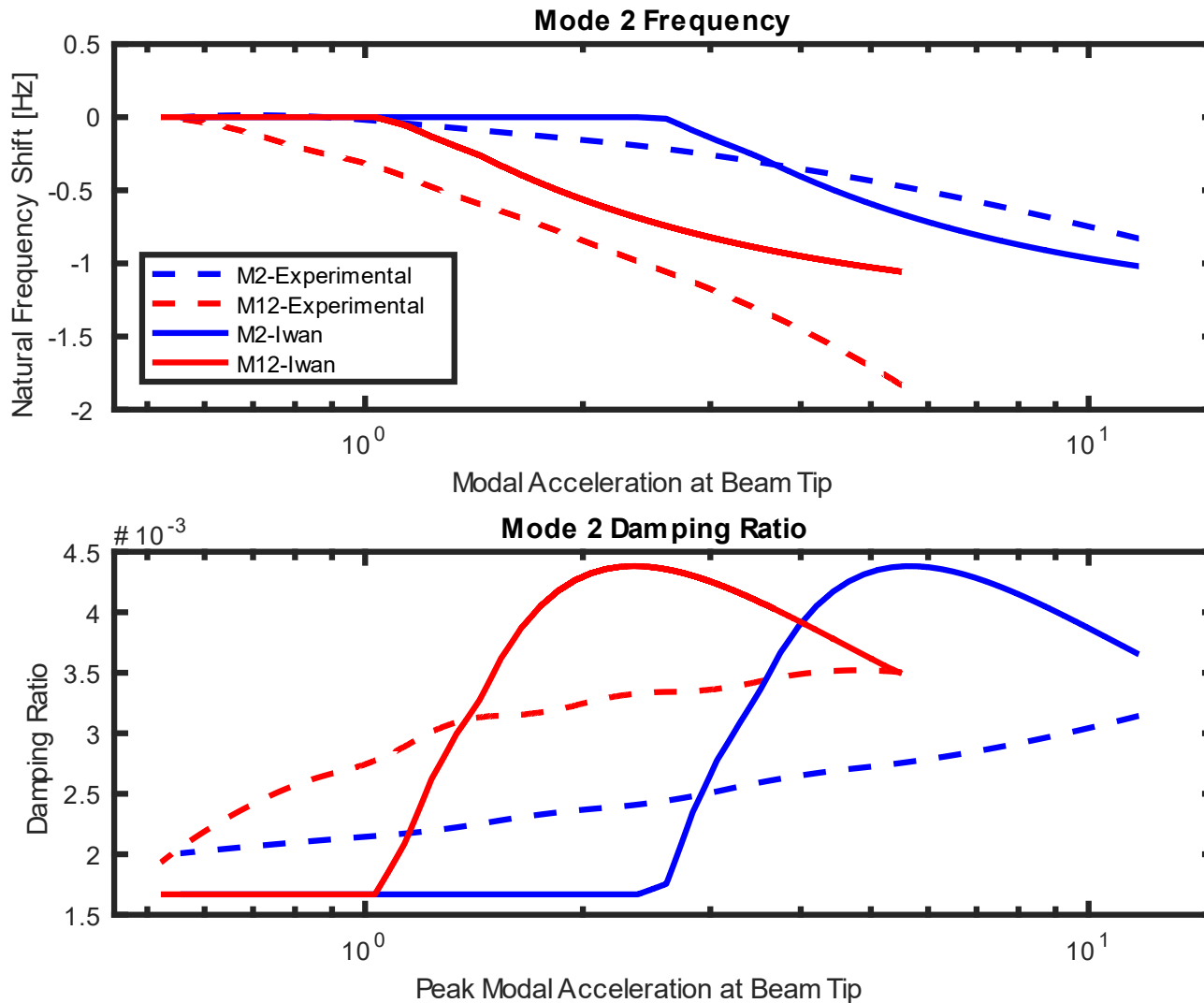
Model directions of slip with Iwan Joints
(Radial and Tangential on Cylinder – Plate Interface)

Iwan Mode 2

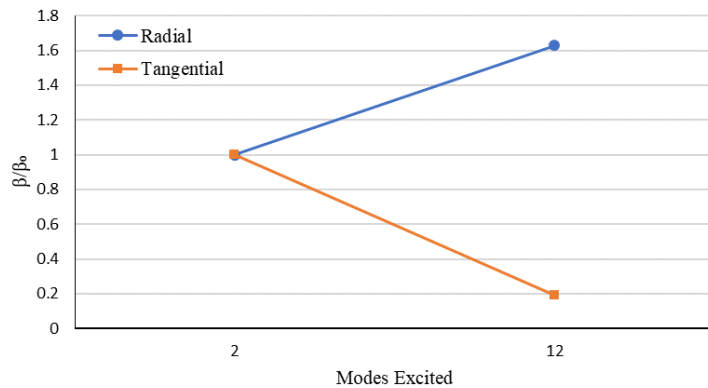
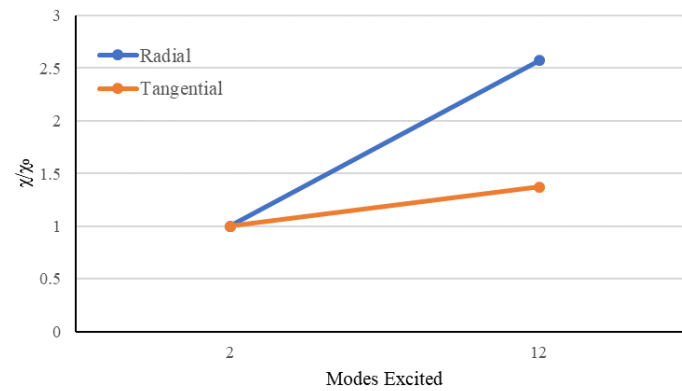
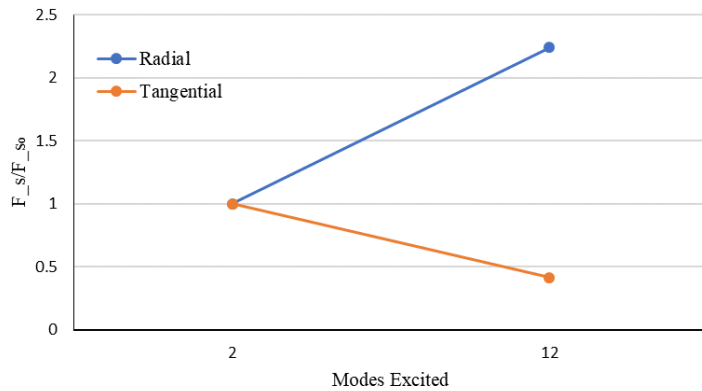
Joint	F_s	K_T	χ	β
Radial	2104	2.26E+05	-0.237	5.51
Tangential	0.199	2.15E+12	-0.692	9.37



Iwan Mode 2



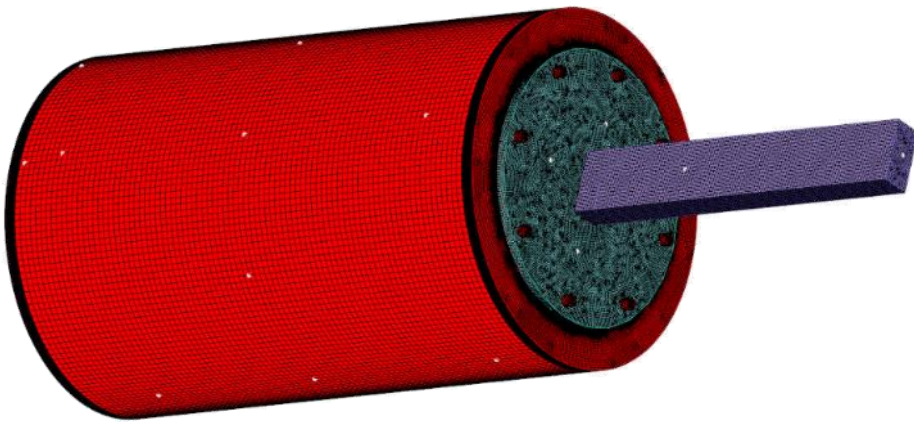
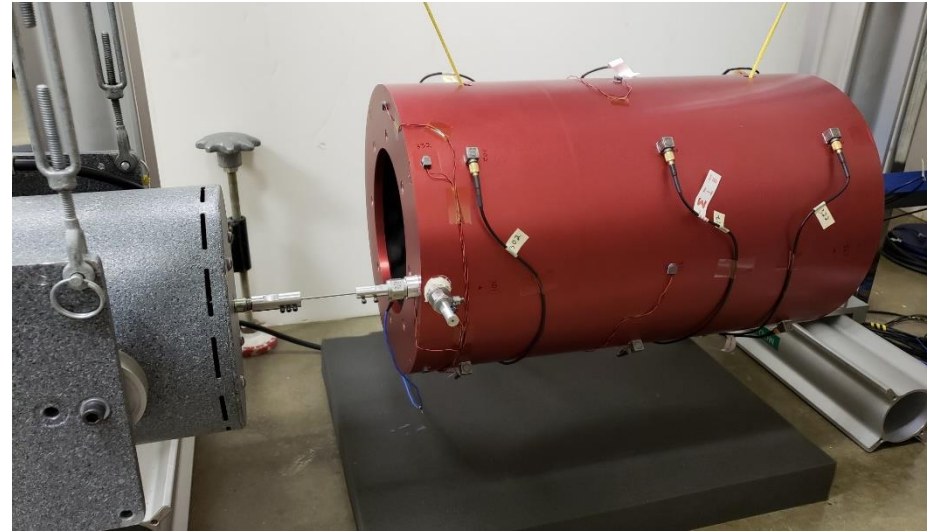
Iwan Mode 2



Iwan parameters for each excitation scaled against excitation of mode 2

Closing Remarks

- Mode 2 was found to couple with mode 1 when both were excited using a shaker
- Mode 1 showed a lesser degree of coupling when multiple modes were excited



- Used a high fidelity model to match nonlinear experimental data
- Iwan models, though currently incomplete, depicted the trends from the Hilbert curves



Acknowledgments

- This research was conducted at the 2018 Nonlinear Mechanics and Dynamics Research Institute hosted by Sandia National Laboratories and the University of New Mexico.
- Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

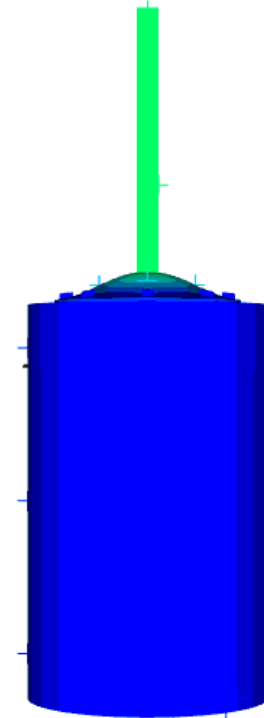
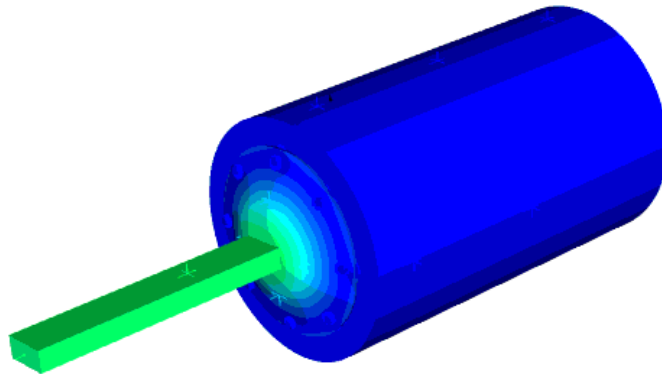


Appendix



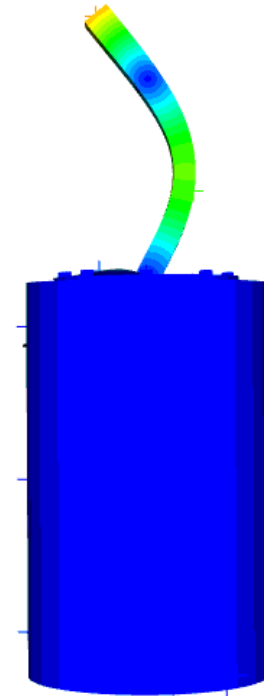
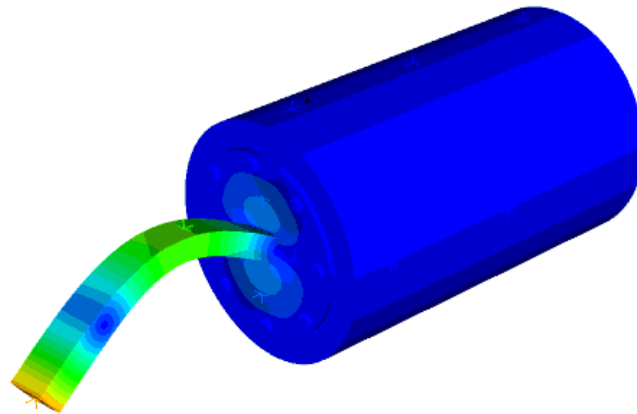
Mode 3

Mode Description	Experimental ω_n (Hz)
Long Plate Drum	548.43



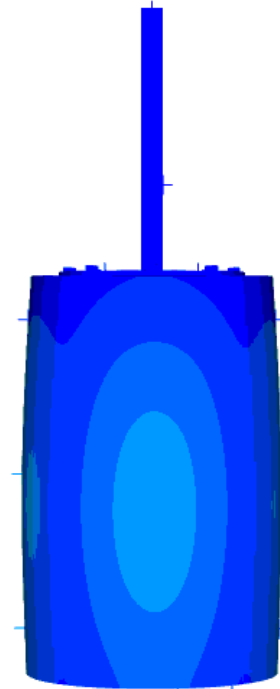
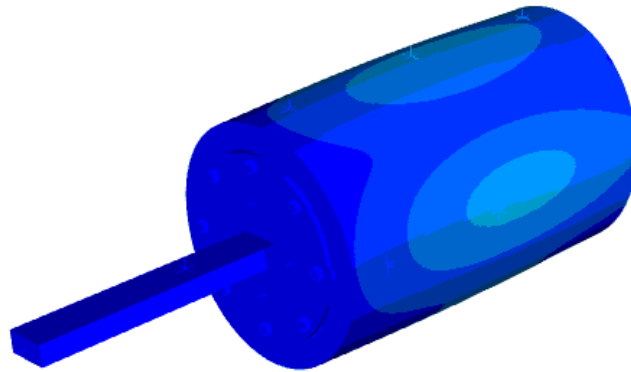
Mode 4

Mode Description	Experimental ω_n (Hz)
2nd Long Beam X	989.47



Mode 5

Mode Description	Experimental ω_n (Hz)
Ovalling	1165.1



Mode 6

Mode Description	Experimental ω_n (Hz)
Ovalling	1165.6

